

✓

✓ ✓

- ✓ 1. See Thompson, look at papers for book, call Pool.
- ✓ 2. Call Hilman
3. Fill out form for Hilman
4. Trip expenses, July, July
5. ~~Handwritten~~ C+C work — Mungel (AFAG)
6. Referee Ninty/Marsall paper

$$\frac{S}{1-p} \quad \frac{1-p}{p} : 1$$

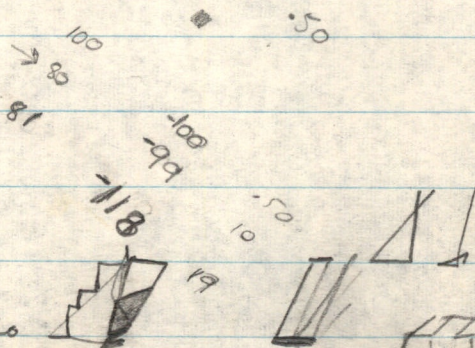
$$\begin{array}{ccccc} S & 0 & S & (1-p)S & -pS \\ 9S & 9S & & 0 & 0 \end{array}$$

If you are indiff. for p & $9S$, then U is linear on money, and you're in. If you aren't...

100	0	-100	0
20	20	-20	-20

2) Berlin

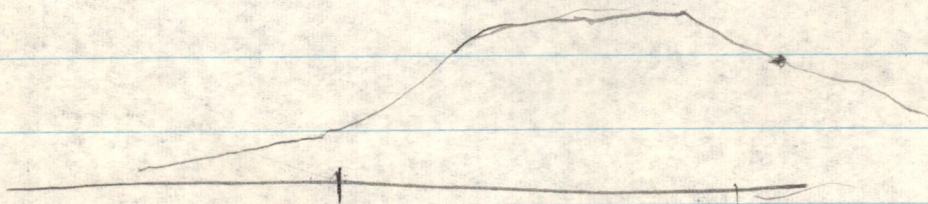
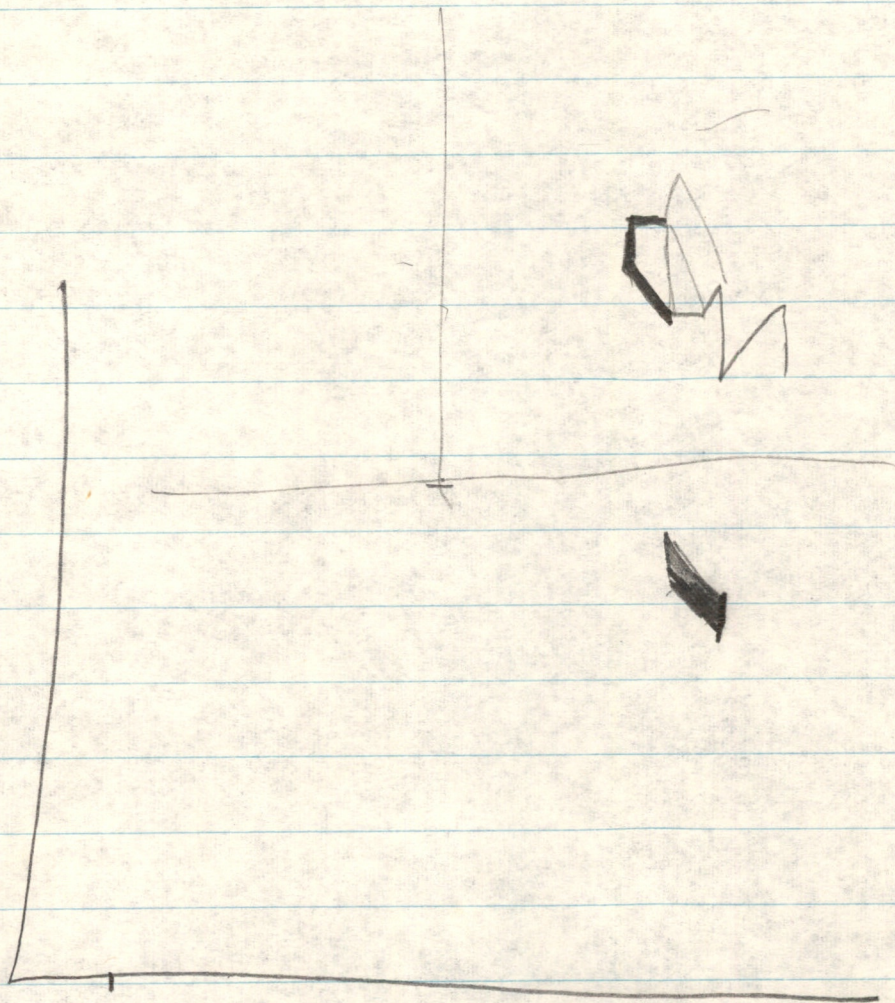
3) Intelligence



Gordon Becker - GE Jumps

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1. My formula is a way of "penalizing" the less reliable dist;
but is it the "best" way, or the way an S really does it?

(e.g., imagine S penalizing it by fixed amount).

2. State axioms that imply (only): S makes prob. judgments in
form $\& "p_1 \text{ is } \geq .5" \text{ or } "p_2 \text{ is between .3 and .7}"$ (i.e. he
rules out $p_1 < .5$, etc.).

These axioms would not require S to obey something Principle.

3. Discussion with Kerner shows the great ambiguity of likelihoods in
cases Intelligence deals with. "If Kerner wanted to achieve this,
what would he do?" "If SU is attacking, what will they do?" Part of
problem is that "hypotheses" refer to unique events in the future; lack of
experience. Part is that they depend on prediction of an opponent's theory.

4. We want indications, for strategic warning, which are highly unequivocal
(every low likelihoods of occurrence if SU is not attacking) with some
significant chance of occurring, and being spotted, if they do attack.

Also, weigh prob. of detecting (+ occurring) vs. cost of detecting. And
weigh ambiguity vs. "lead time" in detection, + cost, etc.

(Vs. "Purple Peril" insistence that indicators have high reliability of
occurrence).

5. Basic heuristic principle of psychoanalysis: 1) A person ~~is~~ behaves not only rationally (w.r.t. some operative wishes in some part of his personality, which may not be consistent with other wishes) but effectively (his expectations are realistic, his choice of instrumental actions efficient, or at least adequate, his judgment of alternative actions available appropriate — so he tends to get what he wants!).

Econs merely predict a man will do what he thinks will be the "best" actions. Analysts use heuristic assumption that he will succeed; hence, that "what he wanted" can be reliably inferred from "what he got," in all its objectively observable and subjectively experienced aspects.

(Like Communists). A better approach for finding effectiveness but non-obvious "wishes" and segments of rationality in complex conduct (Analysts are more ambitious in trying to understand complexities of behavior; hence, more important to them to discover segments of rationality, which will cut down their search problem). [BUT: compare "Freudian typing errors" to ^{text-frequencies,} "Freudian speech errors"; consider influence of keyboard structure, and physical facility.]

6. Axioms are a tactical device for getting people to behave the way you think they ought to behave. If you can derive the desired behavior from axioms that they can be predicted to regard as "reasonable" (i.e. descriptions of the way they want to behave), and if they put a high premium on consistency in their beliefs and ~~hence~~ between their actions and ~~beliefs~~ ideals, then you can create mental tension ("cognitive dissonance") whenever they can be shown to be behaving inconsistently with the axioms. At this point, they will be tempted to disavow the axioms; this is the test of their plausibility and your authority.

7. Theories of choice tell people to behave in a certain way (depending on certain measurements of their beliefs & values). Most theories tell you to ignore certain distinctions about "confidence," "reliability," etc. ~~But~~ People who ~~take~~ make these distinctions and act on them, anyway, will often violate the theory; the theory may seem "reasonable," yet in these cases it seems more unreasonable to ignore the distinctions than to disobey the theory. It is not good advice for those cases.

8. Usefulness of formula $pY_{\text{int}} + (1-p)[\alpha Y_{\text{max}} + \gamma_{\text{min}}^{(1-\alpha)}]$ depends on ability to identify & measure (roughly) α and p . These are not just "dummy" variables (0,1); certain behavior can be explained by intermediate values of α & p that cannot be explained by any of the "component" theories corresponding to (0,1) values for α or p . (Chapman's results). But even as dummy variables, if they can be measured "independently," they indicate areas of application of different "components."

[Illustrate with dichotomy.]

[Show that "irrationality" doesn't matter "much," if true.]

9. You have to look at the date or bibliography of any paper rejecting numerical probes and ask yourself, "Would he have written this way if he had read Savage?" [Part of the time, I would guess "yes," because they seem to emphasize what I consider; other times, "No."] [at least, other times their grounds for rejection would not be inconsistent with acceptance of Savage.]

9. Person who "accepts" his own choices that disbelieve the apions is not merely "tolerant" or "undiscriminating"; ~~rather~~ ("not distinguishing between "good" and "bad" behavior"; having a "broad" range of "adequate" behavior). Rather, he changes the Savage apions with being undiscriminating, with forcing him to ignore a learned, perceived discrimination important to adaptation and survival: between the man where he knows little and the man where he knows a lot.

10. Conservative allocates his experimentation resources "prudently," uses them where he has a good chance — or at least, a known chance — of profiting by them: when big money is involved.

[Suppose he doesn't act so conservatively (?) when small money is involved. Then he won't w.r.t. sampling, where second-order, differences are at stakes: so "conservative" might be more willing to observe in ambiguous situation, if cost is small. CHECK.]

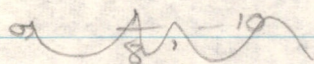
[How expresses role of importance?]

11. Meaning — and meaningfulness — of statements about probs ("probs are known; probs are assigned; probs are reliable, or precise; one acts on probs in a certain way") depends on operational definition of probs.

When a new op def is introduced (Rumsey) previous statements must be re-examined.

Chapman's hypothesis

$$\left(\frac{1}{8}, -10\right) > \left(\frac{1}{10}, -5\right)$$



for $f \geq \frac{1}{2}$

$$C + G - R: (f, n) < (f', n') \Rightarrow P(\varnothing, \varnothing') < \frac{1}{2}$$

i.e. $f < f', n < n'$; or $f = f', n < n'$; or $f < f', n = n' \Rightarrow P(\varnothing, \varnothing') < \frac{1}{2}$

rules out "boldness," or "optimism."

$$f < \frac{1}{2}, f' > \frac{1}{2} \Rightarrow P(\varnothing, \varnothing') < \frac{1}{2} \text{ even if } n > n' \\ (\text{a fort. if } n < n')$$

$$f > \frac{1}{2}, f' < \frac{1}{2} \Rightarrow P(\varnothing, \varnothing') > \frac{1}{2} \text{ even if } n < n'$$

Ex: $f = \frac{1}{3}, f' = \frac{2}{3}, n = 10, n' = 100$

$$\varnothing = \left(\frac{1}{3}, -10\right) < \varnothing' = \left(\frac{2}{3}, 100\right)$$

$$\left(\frac{1}{3}, -10\right) > \left(\frac{1}{3}, -100\right)$$

$$\frac{2}{3}, -10 \quad \left(\frac{2}{3}, 100\right)$$

$$\left(\frac{1}{3}, -10\right) > \left(\frac{1}{3}, -100\right)$$

$$\left(\frac{1}{3}, -10\right) < \left(\frac{2}{3}, 100\right)$$

$$\left(\frac{2}{3}, 10\right) > \left(\frac{1}{3}, -100\right)$$

$$\left(\frac{1}{3}, -10\right) = \left(\frac{1}{3}, -10\right)$$

$$\left(\frac{1}{3}, -10\right) < \left(\frac{2}{3}, 10\right)$$

$$\left(\frac{2}{3}, 10\right) > \left(\frac{1}{3}, -10\right)$$

$$\frac{5}{6}, 10 < \frac{5}{6}, 100$$

$$\frac{5}{6}, 10 ? \frac{2}{3}, 100$$

$$\frac{1}{3}, -10 > \frac{1}{6}, -100$$

$$\frac{2}{3}, 10 < \frac{5}{6}, 100$$

$$\frac{1}{6}, -10 ? \frac{1}{3}, -100$$

$$\frac{5}{6}, 10 = \frac{5}{6}, 10$$

$$\left(\frac{5}{6}, 10\right) > \left(\frac{2}{3}, 10\right)$$

$$\frac{2}{3}, 10 < \frac{5}{6}, 10$$

Problem: how to verify — make operationally meaningful — statements such as "I think $p(x)$ is between .3 and .8, but I can't assign probs in that interval." [Note similarity to confidence interval.]

Perhaps statement means: My interval estimate is produced by a certain process $\Rightarrow$... "How do you act on a confidence interval?" Savage asks. EASY; my rule.]

A snapshot op. test is: How do I expect a large class of events to which I assign prob = $\frac{1}{4}$ to turn out? (see Knight, Good; how to judge a predictor & reward him).

Define "practical certainty" w.r.t. a class of problems (i.e. given "typical actions" or whole set of actions, and scale of regrets).

What probs, or differences in probs, do you tell computer to ignore? What rounding-off error is "acceptable"?

Choice of minimax, Bayes criterion, etc. as a "~~meta~~^{super}-criterion problem." i.e., having selected "appropriate" indices of "payoff" given ^{"relevant"} events X, Y, Z ("relevant" events will depend on actions considered, payoff indices chosen, and predicted payoffs — this will emerge in detail from analysis, but a preliminary "cut" must shut down number of contingencies considered) — i.e. "sub-criteria" — what criterion do you follow when these "conflict"?

"Events" or "states of nature" are the facts "relevant" to the confident prediction of outcomes: the "facts" about which one is uncertain.

Bayes Theorem: does it show how the strength of one's initial beliefs affects the relative influence of initial beliefs/sample info upon ~~post~~ post-sample beliefs? Or does it treat all prior beliefs expressed by the same "odds" ~~as~~ ~~being~~ (e.g., 1:1) as having ~~an~~ equivalent effect upon post-sample beliefs, given the sample results?

Consider ~~these~~ beliefs "based upon" (affected by?) ~~a~~ a small sample — vs. beliefs based upon a large sample — as ~~basic~~ ~~for~~ a priori probs preceding a new experiment (involving, say, intermediate-size sample).

To discover "experts" (advisors) "true" subjective probabilities, you could offer him bets and measure them — if he obeys Savage axioms ^(a) (assuming he doesn't have a motive for deceiving you, or he doesn't know you are observing his bets, or (c) the stakes you offer outweigh his motives for misleading you.

Consider Kelly/McCarthy/Hlood/Marschak proposals for "incentive pay" to forecasters as a system of bets offered to the forecaster intended to measure his probs; I suspect they assume both (1) he obeys Savage axioms, (2) his utility function is known and is arbitrarily specified by these authors (linear on money? Kelly gives an example of logarithmic utility), (3) assumption (c) above.

(2) could be replaced by starting from scratch to measure both probs & utility; but once certain probs & payoffs had been "calibrated," you could speed up process of measuring new probs.

But how do these schemes work when ^(a) expectations are ambiguous, and ⁽²⁾ expert doesn't obey axioms? You would like to know (1), and the "range of ambiguity". Design system of "bets" to get him to reveal ambiguity.

Their problem: give him "bets" such that his expected utility will be maximized if he "chooses to report" his true subjective probs. Select the advisor whose true probs are "more reliable" (by suitable test: a separate problem.) (You can't "pay" forecaster to be right; by paying him amounts contingent on the outcomes of the events he is ~~deceiving~~ predicting, you can only motivate him to ~~try~~ be honest (and perhaps, under certain conditions, to "try hard to be right").

The choice of experts, and reliance upon an expert, reflects the belief that one man's betting behavior may be much ~~better~~ more successful, profitable, than another's. Thus, the "test" or "verification" of ~~A's subjective~~ the "goodness" of A's subjective probabilities — his expertise — is whether his achieved utility is, in the long run, higher than it would have been if he had chosen a different pattern of bets among the bets offered to him.

To determine this, we must know A's utility schedule ~~and his~~ ~~understanding~~ (which assumes that he must have obeyed the Savage axioms in some situations, allowing us to measure his utility scale) and his understanding of the bets (defined by payoffs associated with specified events, not probs) available to him. (Compare my proposition to Manktelow's in "Why 'Should' Bayesianism...", where he omits these conditions and tries to measure "value of obeying axioms".)

$$\left(1 - \frac{1}{1+w}\right)x + \frac{1}{1+w}(-wx) = x - \frac{x}{1+w} - \frac{wx}{1+w} = \frac{x+wx-x}{1+w} - \frac{wx}{1+w} = 0$$

	B	C
Bit	wx	$-x$
Don't		

$$\frac{x}{1+w}$$

$$wx$$



Smith

	B	C
Bet	x	-wx
Don't	0	0

$$p(C) = \frac{1}{3} \Rightarrow [\text{Accept} \Leftrightarrow w \leq 2]$$

"Bet" means "accepting a bet at odds ^{on B} w:1 on B against C" ($w(B:C)$)

w is the l.v.b. of odds at which Bob will accept; essentially,

$$w = w \Rightarrow \frac{1}{1+w} \geq \text{prob}(C). \quad \text{e.g., if } \text{prob}(C) = \frac{1}{3}, \text{ Bob will accept any bet at}$$

2:1 or "better" (1:1, $\frac{1}{2}$:1, etc.) i.e., $w \leq 2$.

$$w(B:C) = 2.$$

	B	C
Offer		
Bet	-Wx	-x
Don't		
Reject	0	0

$$\text{if } x=1, p(C) = \frac{1}{3} \Rightarrow [\text{Offer} \Leftrightarrow W \geq \frac{1}{2}]$$

"Offer" corresponds to "Bob's offering odds W on C against B" ($W(B:B)$)

Ω is g.l.b. to values of W for which Bob will offer bet:

$$\Omega = W \Rightarrow \frac{W}{1+W} \geq \text{pr}(C)$$

$$\text{e.g. if } \text{prob}(C) = \frac{1}{3}, \Omega = \frac{1}{2}$$

Bob will offer bets on C at 1: $\frac{1}{2}$ (2:1 on C) or "better" i.e.

1:1, 1:2, 1:3 on C (^{offer} or 1:1, 2:1, 3:1 on B)

"If reasonable, $w \leq \Omega$ "; i.e. greatest w for which Bob will accept bet on B at $w:1$ (3:1) $\leq$ least W at which he will offer bet on C at 1: $W(B:C)$ or $\frac{1}{W}:1(B:C)$.

$$p x + (1-p)(-wx) = p(Wx) + (1-p)(-x)$$

$$p(x - Wx) + (1-p)(x - wx) = 0$$

$$px - pWx + x - px + pw x = 0$$

$$p x (w - W) + x = 0 = p(w - W) + 1$$

$$p = \frac{1}{W - w}$$

$\frac{1}{2}$
 $\frac{16}{17}$
 $\frac{9}{2}$

Suppose that rule was: Accept if $w \leq 3$
 Offer if $w \geq \frac{1}{2}$ i.e. if $\frac{1}{w} \leq 2$

	B	C		B	C	2:1	4:1
Offer	$\frac{1}{2}$	-1	accept	1	-3	$\frac{2}{3} \times \frac{4}{5} = 1.1$	2:1
Don't	0	0	Don't	0	0	$\frac{1}{2} \times \frac{2}{3}$	0

3:1 1:3 Accept if $w \geq w$ Offer if $w \leq \frac{1}{2}$
 i.e. if $p(B) = \frac{1}{2}$, $w = W = 1$ Accept if $w \geq 1$, Offer if $w \leq 1$
 my rule: accept if $w \geq 2$, Offer if $w \leq \frac{1}{2}$

$w < \frac{1}{2}$

$w < \frac{1}{W}$

$w:1 < \frac{1}{W}:1$

$\frac{W}{1+W} < \frac{1}{1+W}$

B	C	B	C
W	-1	1	-W = $\frac{1}{W} - 1$
0	0	0	0
1	$\frac{1}{W}$	$\frac{-W}{1+W}$	$\frac{1}{1+W}$
0	0	0	0

B	C	B	C	2	-1
St	x	-wx	Bt	-x	+wx
Don't	0	0	Don't	0	0
1	-W	-1	+W	1	-2
0	0	0	0	0	0

How do we reflect "ignorance," "lack of confidence," in our behavior?

Bayesian: by ^{acting "as if" we assigned dist with} high variance, e.g. uniform probs. This does "account" for some discriminatory behavior.

But how distinguish case of "ignorance" from ^{or a die} betting on Heads or Tails?

Bayesian: You can't, without violating some axiom — which would be foolish.

But maybe in this sort of case it's not so foolish to break an axiom; maybe people discriminate, and want to, even though it leads them to violate axioms.

In using Bayes' Theorem, won't "confidence" be ...

Smith doesn't explain under what conditions people might ~~then~~ have $w < \Omega$. if then is this likely; is it "reasonable"? Why might it be preferred? Is there a basis for considering it unreasonable? (Kenny, etc.?)

How does interval ~~from~~ $\Omega - w$ vary? What is significance of size? How does person choose w.r.t. it?

What does Bob's behavior do to the axioms?

Is it perhaps "unreasonable" to allow $\alpha > \frac{1}{2}$?

Can you "make book" against a minimaxer?

How does Bob determine his betting odds? What if he must choose to bet? (Will he be inconsistent? Won't betting odds reflect "best guess" as well as limits of range?)

B C
 Bit W -1

0 0

.70

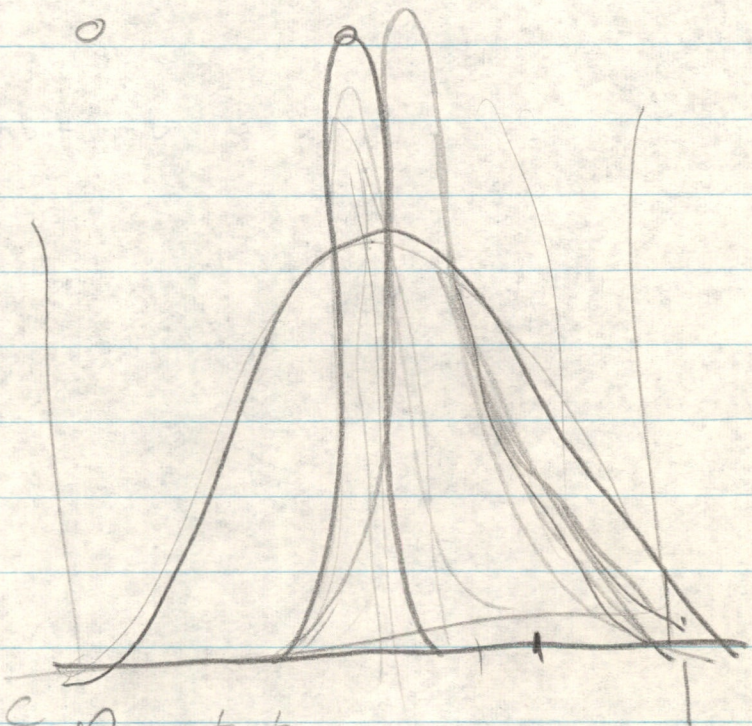
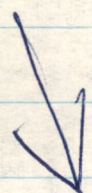
-1

~~W~~

.60

$W \leq 0$ 0

~~W~~



$\frac{1}{3}$

B

C 0

$\frac{1}{2}, \frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

Bit on B if $W > 1$ (I, II, III)

I

W

-1

Bit on C if $W < 1$ (III, II, I)

II

0

0

Indifferent if $W = 1$ $(I = II = III)$

III

$-W$

1

$\frac{2}{3}, \frac{1}{3}$

Never prefer II.

Bit on B if $W > \frac{1}{2}$

Bit on C if $W < \frac{1}{2}$

Indiff if $W = \frac{1}{2}$

$$I \quad p \cdot W + (1-p)(-1) = pW - 1 + p$$

$$p \cdot 1 + (1-p)(-W) = p - W + pW$$

I	W	-1	W	-1
II	0	0	1	-W
III	-W	1	-W	1
IV	-1	X	-1	W
V	1	-X		

$$p \cdot (-W) + (1-p) \cdot 1 = -Wp + 1 - p$$

$$p \cdot (-1) + (1-p) \cdot W = -p + W - pW$$

III ~ Betting against B (at W:1 against B)

~ Betting on C (at W:1 on C)

Correct I	W	-1	Accept (take) a bet <u>on</u> B at W:1 <u>on</u> B
II	0	0	
III	-W	1	Offer a bet <u>on</u> B at W:1 <u>on</u> B (i.e. ^{be willing to} offer to <u>take</u> a bet <u>on</u> C at 1:W <u>on</u> C)

I want (at least) 2:1 on B (before I'll bet on B; i.e. I will choose

$$I \Leftrightarrow W \geq 2.$$

I'll give (no more than) 2:1 on B (if you want to bet on B; i.e. I will

choose III $\Leftrightarrow W \leq 2 \rightarrow$ "I want at least 1:2 on C

$$\Rightarrow p(B) = \frac{1}{3}$$

$$w \quad -1$$

$$p \cdot w + (1-p) \cdot (-1) = pw - 1 + p$$

$$1 \quad -\frac{1}{w}$$

$$p \cdot 1 + (1-p) \left(-\frac{1}{w}\right) = p - \frac{1-p}{w}$$

"Nothing is better than 5:3" Interpret (Rational if a description of max odds to offer(?) How about as nothing min odds to accept, i.e. accept a bet on anything if better than 5:3 is offered, e.g. 3:1?

Bayes behavior, like minimax, can lead you to act under ignorance as if you were almost certain: e.g.

	R	Y	B		5:3 3:1
					R Y
I	1	1	0	II	1 0

I p. II.

Will Bayesians bet heavily that $\frac{2}{3}$ of the time, in the long run, he will win with bets on I?

Some differences in taste are very strong.

There are situations in which I feel strongly it is foolish to obey the Savage postulates. But would I want to rule that as irrational? i.e. would I subscribe to an axiom that forbid that?

$$2 - \frac{2}{1+0} = \frac{2+2 \cdot 0}{1+0}$$

similar to Smith: my old x_a, x_b, y_a, y_b

	x	$\bar{x}$	x	$\bar{x}$
Utilities known, I	1	0	III 0	1
does not. II	a	a	IV b	b
	x_a		x_b	
	a	a'	b	b'

$$\sim [x_a < y_a \text{ and } x_b < y_b]$$

$$\begin{aligned} a &\equiv pr(x) \\ b &\equiv pr(\bar{x}) \end{aligned} \left. \vphantom{\begin{aligned} a &\equiv pr(x) \\ b &\equiv pr(\bar{x}) \end{aligned}} \right\} \text{Bayes (money)}$$

$$\text{Savage: } a + b = 1. \quad (\text{Utilities})$$

"Rule of reason": (?) $a + b \leq 1$.

Suppose $a + b > 1$.

Then if one had a rule, "Buy I if 'price' is $\leq a$, and buy Bet III if price is $\leq b$ ", and you were offered I for a and III for b you would buy both, so that if x occurred, you would net $1 - (a + b)$ and likewise if $\bar{x}$ occurred. < 0 in either case.

"Buy I if price is $\leq a$ " $\approx$ (if price is) accept I': $\begin{matrix} x & \bar{x} \\ 1-w & -w \end{matrix}$ if $w \leq a$
Buy III if price is $\leq b$ $\approx$ accept III': $\begin{matrix} x & \bar{x} \\ -w & 1-w \end{matrix}$ if $w \leq b$.

First is: Bet on x at $\frac{1-a}{a}$ or better ($a' < a$)

Second: Bet on $\bar{x}$ at $\frac{1-b}{b}$ or better ($b' < b$)

$$a+b \leq 1 \quad \text{if } a+b \leq 1, \quad \frac{1-b}{b} = \frac{a}{1-a}$$

~~2~~

$$a, b \geq 0, \quad \frac{a}{1-a} = \frac{1-b}{b}$$

$$A \geq \frac{1}{B}$$

$$b \leq 1-a$$

$$\frac{1-b}{b} \geq \frac{a}{b} \geq \frac{a}{1-a}$$

$$a \leq 1-b$$

So: Bet on X at $A:1$ or better, where $A = \frac{1-a}{a}$

Bet on $\bar{X}$ at $B:1$ or better, where $B = \frac{1-b}{b}$

$$\text{Rational: } B \geq \frac{1}{A} \quad BA \geq 1$$

Suppose: $B = \frac{1}{A}$. EX: a) $A=1 \Rightarrow B=1 \Rightarrow p(X) = p(\bar{X}) = \frac{1}{2}$

$$BA=1$$

$$b) A=4 \Rightarrow B=\frac{1}{4} \Rightarrow p(X) = \frac{1}{5}, p(\bar{X}) = \frac{4}{5}.$$

Kenny:	E	$\bar{E}$	E	$\bar{E}$
I	$(1-q)S$	$-qS$	1	0
			q	q

If I accept I, I am betting on E, and giving odds of $q:(1-q)$.

(I am "getting" odds of $1-q:q$, or $\frac{1-q}{q}:1$?)

If $\text{pr}(E) = C$, "if we bet on E, we must offer $C:(1-C)$ odds"

i.e. we "must" "get" $1-C:C$, or $\frac{1-C}{C}:1$ at least

We must offer no better than $C:(1-C)$ odds.

If opponents' probs and utilities are exactly like ours, and he is reasonable, he will demand at least $C:(1-C)$ odds.

So, if we bet at all, it must be at $C:(1-C)$ odds, which gives each of us 0 expectation.

Kenny assumes: each will take any bet with positive "expected" utility.

Assumption: we can bet only with opponent exactly like us (and certainly reasonable). We can't get better than expectation of 0 utility (since he won't accept a bet that has negative expectation of utility — IS THIS BASIC NOTION OF REASONABLENESS? Suppose a "conservative" will "accept" (and ignore) gambles only if their (best guess) expected utility is $> \epsilon > 0$? where $\epsilon = \epsilon(p, \alpha)$

Savage, et al, assumption: there are always odds ^{on E} (2:1, 3:1, etc.)
 at which you will be indifferent between betting on or against an
 event E. e.g. (if $p(E) = \frac{1}{4}$) you might be indifferent between
 3:1 on E or 1:3 against. ~~the~~

$\frac{1}{4}$	$\frac{2}{4}$
3	-1
0	0
-3	1

$p(E)$ determines an upper bound to odds B will ~~accept~~ offer; but not
 necessarily the best upper bound.

To assign a value to $p(E)$, there must be some event for which he
 will go up to p . ($p=1$)

$$\frac{1}{2} \quad \frac{1}{2} \quad 5 \quad 10$$

$$p X_{\text{ext}} + (1-p) X_{\text{min}} - X'_{\text{ext}} > 0$$

$$X_{\text{ext}} > \frac{X'_{\text{ext}} - (1-p) X_{\text{min}}}{p} = \frac{10 - 2.5}{\frac{1}{2}} = 17$$

My hyps: p 's ("actual" intuitive degrees of belief) may satisfy ~~Thomson's~~ axioms (e.g. Ax I, Anti-symmetry) but r 's, betting quotients, may not, without r 's being unreasonable: because "upper and lower" betting quotients may differ (if they were equal, then $r_i = p_i$, and r 's must obey Axioms if p 's do).

~~§~~ Rule of reason: $r_i \leq p_i$ (not, $r_i = p_i$).

① Suppose $r_i < p_i$. ② Then $(1-r_i) > (1-p_i)$

~~Reason requires:~~ ~~Let~~ R_i stand for betting quotient for $\bar{H}_i$.
and q_i be prob($\bar{H}_i$)

Reason requires:

③ a) $q_i = 1-p_i$

④ b) $R_i \leq q_i = 1-p_i < (1-r_i)$ (by assumption above)

⑤ Suppose $R_i < q_i$. ⑥ Then $R_i < (1-p_i) < (1-r_i)$

⑦ Now suppose $r_j = p_j$, $R_j = q_j = (1-p_j) = (1-r_j)$ (unambiguous)
and suppose ⑧ $p_i = p_j$, ⑨ $q_i = q_j$

Nevertheless, it ^{will be true} ~~is possible~~ that:

⑩ $r_i < r_j$ (by ⑦) and ⑪ $R_i < \del{R_j} R_j (by ⑤, ⑧, ⑨)$

In fact, $R_i < R_j < (1-r_i)$

$r_i < p_i < (1-R_i)$ (by ①, ⑤, and: $(1-R_i) > (1-q_i) = p_i$) and

$R_i < q_i < (1-r_i)$ (by ⑤, ⑥, and: $(1-r_i) > (1-p_i) = q_i$)

A) Now, suppose $r_i = r_j$, $R_i = R_j$, but $r_i \notin (1-R_i)$, $r_j \notin (1-R_j)$

What can we say about p_i and p_j ?

$$r_i = r_j < p_i, p_j < (1-R_i) = (1-R_j)$$

but is $p_i > p_j$, or $p_j > p_i$ or ~~or~~ $p_i = p_j$? We don't know.

B) On the other hand, suppose $r_i < (1-R_i)$ ~~and~~ (so that $R_i < (1-r_i)$)
and $r_j = (1-R_j) < r_i$

Then $p_i > p_j$

C) Similarly, if $r_j < (1-R_j) < r_i$, then ~~$p_i > p_j$~~ $p_i > p_j$

And if ~~$r_i < (1-R_i) < r_j$~~ $r_i < (1-R_i) < r_j \leq (1-R_j)$, then $p_i < p_j$.

D) What if: $r_i < r_j < (1-R_i) < (1-R_j)$?

$p_i < p_j$? We don't know. It is arbitrary to assume that

$$p_i = \frac{r_i + (1-R_i)}{2}$$

2

Smith assumes person will act "as if" he draws p_i

at random between r_i and $(1-R_i)$ when he has to decide in cases like (A) or (D).

I think "choice" of p_i between (r_i) and $(1-R_i)$ will depend on payoffs.

E_i $\tilde{E}_i$

$\bar{r}_i$ is the l.u.b. of r_i' 's for which $I \geq II$.

I 1 0 $\bar{r}_i$ is utility such that X is indifferent between

II $\bar{r}_i$ $\bar{r}_i$ I and II; for $r_i' > \bar{r}_i$ X will prefer II.

$\bar{r}_i$ is "the most that X will pay for the gamble I"

or "the worth to X of a ~~lottery~~ prize of 1 on E_i ".

$\bar{r}_i$ is the largest utility for which
 $I' \geq III$.

	E_i	$\tilde{E}_i$
I':	$(1-\bar{r}_i)$	$-\bar{r}_i$
III	0	0

I' is "a bet on E_i at odds of $(1-\bar{r}_i): \bar{r}_i$ on E_i "

So, by def. of $\bar{r}_i$, $(1-\bar{r}_i): \bar{r}_i$, or $(1-\bar{r}_i): 1$, are the best or worst odds at which X will bet on E_i . If $r_i' > \bar{r}_i$, so that odds on E are less than $1-r_i': 1$, X would prefer not to bet at all than to bet on E at r_i' these odds.

Does this mean that at $\frac{(1-r')}{r'}: 1$ on E, where $r' > \bar{r}_i$, X would prefer to bet against $\tilde{E}$ (on $\tilde{E}$) rather than (a) on E, or (b) not betting? Helman, Skimming, Kenney (Rausy, de Finetti, Savage) assume Yes. C.A. Smith, I (and Koopman?) (and Good, Henning, H&L?) assume No, not necessarily. This does not seem required for "Coherence"; although it is required for conformity to axioms (other than Koopman's?).

	E	$\tilde{E}$	
IV	0	1	
V	$\bar{R}_i$	$\bar{R}_i$	$\bar{R}_i$ is the ^{utility} greatest amount for which X is indifferent between IV and V. $\bar{R}_i$ is the <u>l.o.b.</u> of R_i 's for which $IV \geq V$.
IV'	E_i $1-\bar{R}_i$	$\tilde{E}_i$ $1-\bar{R}_i$	
III	0	0	

$\bar{R}_i$ is the greatest utility for which $IV' \geq III$.

IV' is a bet on $\tilde{E}$ at odds of $(1-\bar{R}_i) : \bar{R}_i$ on $\tilde{E}$, or odds of $\frac{1-\bar{R}_i}{\bar{R}_i} : 1$ on $\tilde{E}$.

By def of $\bar{R}_i$, $1-\bar{R}_i : \bar{R}_i$ on $\tilde{E}$, or $\frac{1-\bar{R}_i}{\bar{R}_i} : 1$ are the worst odds on $\tilde{E}$ at which X will bet on $\tilde{E}$. If $R_i > \bar{R}_i$, so that $\frac{1-R_i}{R_i} < \frac{1-\bar{R}_i}{\bar{R}_i}$, X would prefer III to IV', i.e. he would prefer

not to bet.

[NOTE r_i, R_i here are not equivalent to the "betting quotients" in earlier discussion] [before double line]

Question: what is the relation between $\frac{1-\bar{F}_i}{\bar{F}_i}$ and $\frac{1-\bar{R}_i}{\bar{R}_i}$, between the "least odds ^{on E} at which he will bet on $\tilde{E}$ " and "the least odds ^{against E} at which he will bet against E "?

Note: $I' \quad E \quad \tilde{E}$
 $(1-r_i) \quad -r_i$

accepting a ~~bet~~ but on ~~#~~ E getting odds of $\frac{(1-r_i)}{r_i} : 1$ on E is the same as offering a bet as offering a bet on $\tilde{E}$ giving odds of $r_i : (1-r_i)$ on $\tilde{E}$, or $\frac{r_i}{(1-r_i)} : 1$ on $\tilde{E}$.

Or, as offering a bet against E , ^{giving} odds of $\frac{r_i}{(1-r_i)} : 1$

against E .

So, if $\frac{1-\bar{r}_i}{\bar{r}_i} : 1$ are the worst odds on E (against $\tilde{E}$) at

which I will ^{accept} bet on E (against $\tilde{E}$), by the same token,

$\frac{\bar{r}_i}{(1-\bar{r}_i)} : 1$ are the best odds ^{on $\tilde{E}$} ~~at which~~ I will give in offering a

bet on $\tilde{E}$ (against E).

Def: Let $b_i = \frac{1-\bar{r}_i}{\bar{r}_i}$. Then I will demand at least odds of $\bar{b}_i : 1$ to bet on E , and I will give at most $\frac{1}{\bar{b}_i} : 1$ on $\tilde{E}$, or against E , in offering bets against E .



Def: let $B_i = \frac{1-\bar{R}_i}{\bar{R}_i}$

Then B_i I will demand ~~at least~~ odds of $B_i:1$ or better to accept a bet on $\tilde{E}$, or against E , and $\frac{1}{B_i}:1$ are the best odds I will give ~~against~~ on E , or against $\tilde{E}$, in offering bets on E .

Thus, if $P_i:1$ are odds on B such that $P_i \geq b_i$ ($\frac{1}{P_i} \geq \frac{1}{b_i}$)

- 1) I will accept a bet on E_i at $P_i:1$ on E .
- 2) I will offer a bet against E_i at $\frac{1}{P_i}:1$ against E (i.e. $\frac{1}{P_i}:1$ on $\tilde{E}$).

If $P_i \geq B_i$ ($\frac{1}{P_i} \leq \frac{1}{B_i}$):

- 3) I will accept a bet on $\tilde{E}_i$ at $P_i:1$ on $\tilde{E}$
(i.e. I will bet against E_i at $P_i:1$ against E)
- 4) ~~I~~ I will offer a bet at $\frac{1}{P_i}:1$ on E (against $\tilde{E}$).

What is relation between the ^{b_i} ~~highest~~ ^{g.l.b.} lowest odds on B at which I will bet on B and the ^{$\frac{1}{B_i}$} ^{l.u.b.} highest odds on B at which I will bet against B
 $B_i:1$ on $\tilde{E}$ or $\frac{1}{B_i}:1$ on E
 (i.e. the lowest odds on $\tilde{B}$ at which I will bet on $\tilde{B}$) ?

~~Def:~~ To be "coherent": $b_i \geq \frac{1}{B_i}$

or $\frac{1-\bar{R}_i}{\bar{R}_i} \geq \frac{R_i}{(1-\bar{R}_i)}$

If $b_i = \frac{1}{B_i}$, $r_i + R_i = 1$

If $b_i > \frac{1}{B_i}$, $r_i + R_i < 1$.

Let $c_i:1$ be my "true" odds on E_i

Suppose my "best guess" odds on E are $c_i:1$.

(If my "best guess" $p(E) = p_i$, my best guess odds on E are $\frac{p_i}{1-p_i}:1$, so

$c_i = \frac{p_i}{1-p_i}$) So best guess odds on $\bar{E}$ (against E) are $(1-c_i)$

($\frac{p_i}{1-p_i}$ odds on E)

But I may think "true odds" may be anywhere ~~$(c_i:1)$ to $(c_i:1)$~~

from $w:1$ to $W:1$, where $w < c_i$ and $W > c_i$. $w < W$

i.e. odds on $\bar{E}$ may be from $(1-W):1$ to $(1-w):1$.

i.e. odds on $\bar{E}$ may be from $\frac{1}{W}:1$ to $\frac{1}{w}:1$.

I must get at least $w:1$ to bet on E , and I must get at least $(1-W):1$ to bet on $\bar{E}$; i.e. I must get at least odds on E must be at most $\frac{1}{(1-W)}$ before I will bet on $\bar{E}$.

But since $w < W$, $\frac{1}{W} > \frac{1}{w}$, $\frac{1}{1-W} < \frac{1}{1-w}$

Suppose $\frac{1}{1-W} < w$

$$1 < w(1-W) = w - wW$$

$$wW < w - 1 \Rightarrow 1 - w < -wW$$

Then suppose $\frac{1}{1-w} < Y < w$

$$w-1 > wW > 1$$

$$W > 2$$

If odds on E are $Y:1$, I will prefer not to bet on E at $Y:1$ on E and I will prefer not to be against E at $Y:1$ on $\bar{E}$ (

$$\frac{1}{1-W} = w \Rightarrow 1 = w - wW$$

$$wW > w - 1$$

$$\frac{1}{w} = 1 - W \Rightarrow 1 - w = -wW$$

I must get at best $w:1$ to bet on E and I must get at best $\frac{1}{W}:1$ to bet on $\bar{E}$ (against E).
 i.e., odds on E must be at most $W:1$

Suppose $b_i > \frac{1}{B_i}$. Then consider $m_i \rightarrow b_i > m_i > \frac{1}{B_i}$

At $m_i:1$ on E , I would not accept bet on E ; I would prefer not to bet. And I would prefer not to bet than to accept bet against E .

$$\frac{1 - \bar{F}_i}{r_i} < m_i < \frac{\bar{R}_i}{1 - \bar{R}_i}$$

$$\begin{array}{l|l} 1 - r < mr & m - Rm < R \\ 1 < mr + r = r(1+m) & m < R(1+m) \\ \frac{1}{1+m} < r & \frac{m}{1+m} < R \end{array}$$

~~$m_i r_i > 1 - r_i$~~

1	0	1	0	
r_i	r_i	I $\frac{1}{3}$	$\frac{1}{3}$	i II
0	1	II 0	1	$(1 - \frac{1}{2}) - \frac{1}{2}$
R_i	R_i	III $\frac{1}{3}$	$\frac{1}{3}$	i IV
0	0	IV $\frac{1}{2}$	$-\frac{1}{2}$	
		V 0	0	
		VI $\frac{2}{3}$	$-\frac{1}{3}$	i V, VII
		VII $-\frac{1}{3}$	$\frac{2}{3}$	i VI, VIII

$$.2 < p < .4$$

$$.4 < (1-p) < .8$$

$$p_*(a) + p^*(\bar{a}) = 1$$

$$\cancel{p(a)} \rightarrow p^*(\bar{a}) = 1 - p_*(a)$$

$$p_*(\bar{a}) + p^*(a) \geq 1$$

$$p^*(\bar{a}) + p_*(a) \leq 1$$

The fundamental heuristic axiom of economists has always been that their own introspection will ~~provide them~~ inform them of rules of reasonable behavior; that men should behave reasonably; and that ^{often} ~~sometimes~~ they do.

Hypothesis of reasonableness is not naive; A) it is used heuristically, as a hypothesis; B) it is, now, really a bundle of hypotheses, suited to different situations, capable of covering a wide variety of behavior.

It is reasonable to judge and choose an act by its ^{anticipated} consequences. But what if its consequences are not sharply known?

Guidance / description for case of ignorance is inadequate and conflicting and various.

Abstraction is necessary: ignore differences between X and Y; ~~ignore differences~~; notice differences between $(X \cap Y)$ and Z. Ignore certain differences, to focus upon others.

Bayesian theory ignores differences between situations of equal "probabilities"; ignores differences in confidence, ambiguity. OK if it works. But it does not; either normatively or descriptively. And reason it fails, when it does, is related to these differences. Yet they can be accounted for systematically.

(Enlarge failure, to motivate more complex theory; Note: Savage/Rosay deserve credit for inventing the operations that make both their theories & mine meaningful — that disprove theirs and support mine).

Value of info:

My calculations on value of info to "conservative"
 $(p < 1, \alpha < \frac{1}{2})$ assumed that he expected p to be constant after
 info (experiment).

But main effect anticipated may be change in p , not in y_0 .

	A	B
I	-1	1
II	1	-1

Also, "conservative" may act "as if" $p=1$ when

possible losses are trivial: e.g. CHEAP EXPERIMENTATION

$\alpha=0, p=\frac{1}{4}, y_0 = (\frac{1}{2}, \frac{1}{2})$ Value of game = 0.

("regrets"?)

Info: will make $p=1$, y_0 either $(1,0)$ or $(0,1)$, with best guess probs $(\frac{1}{2}, \frac{1}{2})$.

Value (info) = 1.

(Kart/Tintner's "likelihoods" are like prob. dist. over "results of a definitive
 experiment or observation on proportion p of balls in an urn.")

$\frac{1}{2}$	$\frac{1}{2}$
A	A
3	0
1	1

assume 2 balls in urn, observation will show either $(0,1)$ $(1,0)$ $(\frac{1}{2}, \frac{1}{2})$

sample of 2:

3 0

Bayesian: $V(G) = 1.5$

$$V(\text{Info}) = \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 1 + \frac{1}{2} \cdot 0 = .25$$

1 1

Me: $p = \frac{1}{4}, y_0 = (\frac{1}{2}, \frac{1}{2}), \alpha = 0; p \text{ after Info} = 1$

$V(G) = 1$,

$$V(\text{Info}) = \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 0 + \frac{1}{2} \cdot .5 = .75$$

$$V(\text{Info}) = p[.75] +$$

YOUNG DOCTORS

So if $p > \frac{1}{3}$, value of info is greater to Conservative; because

a $(\frac{1}{2}, \frac{1}{2})$ outcome of observation is valuable to him, raising p and "permitting"

him to choose I instead of II. (If $p > .66$ to begin with, say $p = .75$ he would

have chosen II anyway: $V(\text{Info}) = p[\frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 1 + \frac{1}{2} \cdot 0] \leq .25$.

Flexibility

Tintner, Hart imply that value of flexibility does not depend on dispersion of a given prob. dist. but with the expectation that knowledge will improve over time, before a "final decision" must be made.

Associated with "second-order probs" (Tintner, Good) (second-order ignorance; see, Good...). Hence, Hart says, don't "compound probs"; you lose the "datum" that one is uncertain about the prob. dist. of outcomes; which, Hart assumes, means one believes that info could be better. → flexible planning.

(T Marschak & Nelson try to explain flexibility in terms of variance of overall dist, or "primary" dist. Higher-order uncertainty is associated with higher-variance now, but the expectation of lower variance (and perhaps quite different mean) "later" — after more observation.

(Hart still assumes Type II probs exist; but he wants them kept separate). See Stigler, JPE XLVII, 1939 "Production & Dist in the Short Run"

Basic fact: different types of uncertainty tend to be associated with a) different time periods, and b) different sorts of actions. So distinctions are especially relevant to: a) problems of planning, commitment, sequencing of actions; b) decisions to "decide" or "postpone," or to acquire more info; c) decisions between different classes of action (some "unfamiliar," "unusual," "controversial...")

W.r.t. decision tree, compare value of "game" if a definite path through the tree must be chosen initially, with no moves contingent upon later info, with value of "strategy."

(e.g., assume opponent's moves are independent of yours: Nature.?).

See Marschak, *Economica*, Aug 1938

"When 'should' planners construct strategy, instead of 'plan'?"

"In the state of ignorance, all probabilities are equal, but some are more equal than others." (All probs are private, but some are more private than others).

"Men try to be reasonable, and sometimes succeed. So, at least, economists like to assume; it is their main heuristic principle.

As their starting point for investigation, it is their fertile source of ~~guesses~~ initial hypotheses, guesses, tentative advice and prediction

Flexible: a bundle of hypotheses

How does a reasonable man act? (Observe men at their most thoughtful, organized, consciously purposive — as in organizations). Or: Economists ask themselves, and check the answers with their colleagues. & Ingenious trust that introspection — by economists — provides insight into reasonable behavior.

In my article, I stress that "no probs can be measured."

Should have stressed: no precise probs can be measured.

A theory that has no place for interval estimates, for distinction between confidence + ambiguity, cannot describe or advise in many decisions that turn on such distinctions.

PROBLEM: NEED A DECISION RULE FOR ACTING ON
INTERVAL ESTIMATES

(before predictions or advice can be based on such estimates;
to motivate invention of a language for describing such estimates, or tools
for measurement).

Current emphasis on normative aspect; statistical decision theory.

('Stats', too, take as a first guess that what they have been doing is reasonable.

Their payoff fns are vague; but the likelihood fns attached to their
statistical hyps are not "objective frequencies." Hence, at best "objective
payoffs," e.g. with 0,1 weights are well-defined. What should they
decide; and advise. Stats have themselves as subjects; they are the
deciders. Moreover, their "decisions" tend to determine others decisions.

(How should client use their "conclusions"?).

Their distinction between "precisely known" likelihoods and
"ignorance" of hyps. Is their uncertainty about hyps of a different "order" from
their uncertainty about conditional events? Savage: NO; only "more" personal.

Exclusive prescription with the long run ~~may~~ may justify the
prediction: In the short run we shall all be dead.

On Strategy

One problem: likelihood of war, and effect of ~~some~~ policies on likelihood.

"Arms races have always led to war." ^{very true.} But so has the lack of arms races. So has every policy. (And wars have been both temporally and causally preceded by very different security policies. No simple theory of the relation of policy to war (e.g. arms budget) works.

Disarmers like to point to WWI. Arms point to WWII. Both draw valid "lessons" for a certain sort of situation. Unfortunately, the present situation seems to have aspects of both sorts of situation.

The "lessons of history" must be fitted into a rather complex framework if they are not to lead us into great error. Hence, cannot ignore importance of analysis, prediction, test, criticism, inventiveness in constructing models, tests, criteria, alternatives.

If we do not learn to understand war, then in the short run we shall all be dead.

Problem in forming expectations about opponent: conditional probs for his actions given his objectives are not well understood; Hence, the "bearing" of observations of his behavior is ambiguous.

Secrecy.

Neo-Bayesian claim their more formal, explicit inferences from their preferences and beliefs are less arbitrary ("more objective") than the decisions made by Neyman - Pearson + Fisher approaches. Raiffa - Schlaiffer p. vii (Good:)

Surely, to make personal judgments more explicit, more public, is the first step toward making them more "objective," more directly based on evidence, in line with other "reasonable" opinion, non-controllable (Freud: When was I, then shall I go be.'). Enables + encourages evidence being brought to bear on these judgments.

"Objective probs": There exists, available, a body of evidence such that given this evidence any two "reasonable" men would assign virtually the same subj. prob. measure to events. (R+S, p. 5)
Can happen. (Even Fisher accepts such "a priori" probs).

Use of axioms: not so much to "police" preferences (Savage) and "detect inconsistencies" as to generate new probabilities which are acceptable (on reflection) as basis for action, from "given" probs.

To cut my problem down: I assume no further observation possible, no experiments, decision must be made.

"Terminal decisions." (For a statistician, or Intelligence, what to "infer", what to report, how to describe, what to recommend; for Commander, what to do — excluding possibility of "let more info.")

I assert: Internal estimates of probs "really" describe uncertainty. Point is to (a) show the operational meaning of this notion, and (b) show interdependent implications for description, measurement of uncertainty and the decision rules based on it. Certain behavior is ruled out by hyp., including much ruled out by Savage, and — in some circumstances — behavior implied by Savage. Savage behavior is special case, invalid in certain specified areas (my critique of Savage is not merely that he is "slightly wrong" — i.e., "only approximate — part or most of the time).

Games

From a Bayesian point of view, decisions w.r.t. the game tree will take the form specified by VNM only when this corresponds to player's actual subj. probs; i.e. minimizing (or any other allegedly "reasonable" pattern) has implications for player's probs.

Is he willing to bet 1000:1 that opponent will minimize?

Why? Under what conditions?

is Simon: from a normative point of view, you "should" use optimizing analysis if and only if it promises to be "worthwhile."

This is separate from question of whether people do it or not, or which better describes what they do. 1) They may never use it, even when they "should." ~~It~~ (Simon sometimes hints). 2) They may always do it, even when they shouldn't. (less plausible; usual econ assumption). 3) They may not do it when it is not ^{"rational"} worthwhile, and it may rarely or never look worthwhile (on basis of preliminary calculation (Simon asserts; plausibly; but this conforms to "rationality" hyp.)). 4) They may not because it looks not worthwhile, but this may reflect "unreasonable" calculation of costs of rational calculation and possible benefits.

Simon is ambiguous: Do people reject "optimizing" calculation because they are not rational (in some broad sense), or because they are rational and it isn't? What normative advice would he give? How does he want computers to act? In a way that violates broad canons of "reasonableness"?

Flexibility:

Extensive form suggests greater flexibility; you only commit yourself to first move, allowing for possibility of totally unexpected results (conflicting with prior probes; e.g. occurrence of "impossible" outcome, or outcome highly improbable for all hypotheses considered — except, perhaps, for some still considered to have low probability. Or, new alternatives recognized. Or change in payoff values.)

Normal form suggests (though doesn't imply) "rigid" choice of whole strategy, or decision rule, in advance of any action. When this does mean commitment to rule, it is less flexible; presumably implies more careful prior assessment of each action at each step than under first case (this might be worthwhile, w.r.t. costs, speed, and possibility of "stopping to think" at each node during the game, to argue choice with team-mates, to communicate info to central decision-maker and his decision to others.)

Prior choice of decision-rule may allow decentralized "flexible" (i.e., adaptive to info) execution during the game. Might be less scope for this if "extensive" procedure followed; more need for centralized decision-making during the game. Questions of: 2) relative importance & likelihood of "surprises"; 3) costs of pre-planning; c) possibilities and potential advantages of "learning" during the game (as opposed to "planned adaptation.").

Do R+S consider "commitment"?

Question whether you want to examine ~~and~~ ^{formally} analyze each node in a decision tree, looking "forward," as carefully as if it constituted a terminal decision you were now "making" (committing yourself to make).

What if $\alpha > \frac{1}{2}$? Man who chooses ambiguity, against his "best guess" expectation?

(Assume that by a) first exposing him to problem with "important" payoffs, then b) letting him make small side bets).

May violate coherence (as well as axioms) when (only when) there is ambiguity. Is this of practical importance — to him? (Makes him duped of con man, etc.) When Nature is the opponent, will it "make look" "cleverly" against him? Is maximax "sure" to lose? (except in ^{zero-sum} game against ^{informed} clever opponent). (Supposing his "perceived ambiguity" is "realistic" — not "obviously distorted." Savage man "can be had" if his probs are "wrong.")

Assume D must make a decision, somehow. ^{may also} Assume D applies some tests of

"consistency" to the set of his decisions. Some tests imply that D's decision procedure must be formally equivalent to a specific process of resolving conflicts.

→ R+S: whole apparatus is called for to "resolve conflicts" "reasonably" and

"consistently": i.e. for situations where one action is better than another by some "relevant" tests, criteria, objections, aspects or dimensions, contingencies, individual opinions, but worse by others.^{6.14} (Assuming there are all "relevant" to a given — individual or group — decision-maker; an "opponent's" preferences may not be directly "relevant" or potentially "in conflict" — his actions, and his influence upon a joint outcome, may be).

↓ When no "conflict" exists, no problems of the sort of one considering.
↓ Formal similarity of all these types of "conflict"; though no assurance that an acceptable "solution" in one case will look reasonable or applicable in another (e.g. Savage axioms, esp. P2, need not look as reasonable, if at all, in case of conflict between value criteria.

Certainty-equivalents

Knowledge of the relative "best guess" weights assigned to events may not be enough (along with utilities) to determine "certainty equivalents." (cf. equal-weight case; amount one will pay for gamble will depend on "ambiguity.")

Value of info.

R & S, p. 19

Consider implications of getting info which doesn't affect your choice in a given d.p.v.v. "worthless." (It may affect other decisions, later; or affect "value of game." Is latter "worthless" if you ~~are~~ do not have problem of "how much to pay" for game or "which game to play"? Shackle would imply that increased "pleasure of anticipation" is worth paying for, even if no choices are affected. ? What if "confidence" — sense of security — is affected? (similar to Shackle's "pleasure of anticipation").

Info seems valuable in latter case. ? Though would any decisions necessarily be affected? (say you would be taking conservative, or "lamey" action in either case; but info could make you feel better, and this is balanced against chance that it could make you "feel" worse.)

(Suppose conditional probabilities are "objective" — unlike discussion with Dornes — so that bearing of info on one's "feelings" and expectations can't be controlled by decision-maker, nor can content of info (completely), i.e., he can't just "choose to feel better, choose to expect more favorable outcome" without risk.)

Cont.

[If this "improvement in expected payoff to action that would have been taken anyway" is "worth something" to you, then act that way in evaluating info. If not, don't (e.g., does computer have "feelings," "worries"?)
(We just care what computer does, not whether it is happy or frightened.?)

If info has side benefits or value for future problems, show this as part of payoff to experiment. ("final outcome" will then include "having this info, for future problems"; even if it doesn't affect decision in this one.)

R&S definition of "value of info" is thus a narrow one, tied to decision within a given d.p.v.o.

Scientific rule in deciding:

Keep more "objective" (or definite, precise, unambiguous, agreed) data ("premises") distinct in calculation from more ambiguous, subjective, "controversial." - till final "confrontation" for purposes of making decision.

(You can then see whether latter data is "needed" at all for decision; or whether its degree of imprecision "makes a difference"; or how much difference it makes. ~~They~~ Will indicate whether it is worth the trouble to improve latter data or acquire more info; and how much confidence can be put in decision without improving it.

LOGICAL CONSISTENCY

Savage axioms ~~do not~~ of "consistency" do not follow, as R+S imply.
(p. 23, 16)²⁴¹ just from desire to be "logical." Weaker, or conflicting, tests of "consistency" could be just as "logical."

1. This is dominance, not (as they say) P2, or the Sure-thing Principle. 1. 24

2. Continuity of utility over actions w.r.t. each coordinate. Rules out minimax, for which a worsening of the minimum for a given strategy cannot be "compensated" by any improvement in the maximum (or in case of MY RULE: a small worsening of minimum can't always be compensated by a small improvement in maximum. True?)

3. Really a utility axiom, assuming some "objective" probs are known, and available as basis for mixed strats. Minimax & my rule violates. Hence, my rule violates Axi 1, admissibility, if prizes include ambiguous strats and some events have unambiguous probs. Pratt example. 25

Problem of reporting info $\approx$ loss of "sufficient statistics."

The computation of a single, "expected" probability from a range of values is not always a "sufficient" statistic (where prob. dists. over range were ambiguous); range must be reported.

How much would you bet that event (or prob) lies within this range?

1000:1 — to define "possible" hypotheses to begin.

1:1 : 50% confidence interval.

19:1 : 95% conf. interval.

99:1 ; 99% conf. interval. (What is required for decision?)

Test for Arrow/Hurwicz/Debreu case of total ignorance:

Can you divide up space of events into 3 (or more) intervals A, B, C so that S is equally willing to bet for or against $A, (A \cup B), (A \cup C), B, C, (B \cup C)$?

(S won't give 2:1 ^{or, in fact, better than 1:1} on any event or combination of two events: $A, B, C, A \cup B, A \cup C, B \cup C$).

In general, any criterion that would force S to give long odds that event will be confined in a narrow interval (which strict minimax might do) (or minimax might) doesn't seem to reflect behavior associated with "great ignorance" (even though S would shift interval if payoffs were changed in structure). Bayes Laplace might force you to give long odds on a relatively broad interval; but even that might not reflect actual behavior.

(Smith). YOU MAY NOT GIVE LONG ODDS ON ANYTHING, though you might give a little better than 1:1 on some things (?), or might not demand every long odds on some things.

To express ignorance on election, might not be enough to say, "I want at least ^{in utility} 1:1 on Kennedy" and at least 1:1 on Nixon." That would be consistent with 50:50 odds, and with indifference between a) a bet on Kennedy; b) a bet on Heads; c) no bet. (b) and (c) will be indifferent (by assumption on utility states), but (a) and ~~the~~ (b) may not be.

§ To express relative ignorance (between election bet & coin bet) adequately, must say (when true): "I want better than 1:1 on Kennedy and I want better than 1:1 on Nixon."

XX

Vagueness

S&R p. 66: S&R express all prior states of uncertainty by prob. dists, but admit that "we cannot distinguish meaningfully between 'vague' and 'definite' prior distributions" (they all imply S will make long odds bets about some aspects, parameters of prior distribution; e.g. if not about μ , then about $\frac{1}{\mu}$) (even though de Finetti protests ^{EVEN IF HIS BELIEFS ARE VAGUE.} arbitrariness of Laplace approach). And in some cases (p. 67) S "must" specify prior probs quite definitely.

S&R say, they do allow distinguish between dists. which can, or which cannot, "be substantially modified by a small number of ~~the~~ sample observations" ~~p. 66~~ from a particular data-generating process" p. 66; but, if the sample is not taken, they do not allow for this distinction influencing a terminal decision under uncertainty.

I say it does influence decision; in a way that makes it impossible to infer a precise a priori prob dist in the ambiguous case (& that leads to violations of P2, or S&R's Axiom III, maybe I and II). S does not act "as if" he assigned any definite, predetermined weights to the relevant events and their corresponding outcomes: even approximately.

Also affected will be decision to buy an expensive sample, under ambiguity (probs can be affected greatly, but hard to assess likelihood that they will be). However, I don't explore this area fully.

I suspect notion of prior probs is not too important in the two extreme cases: (a) when one's "prior sample" ~~is~~ is very small; confidence low, vagueness or ambiguity high, conf. intervals large; (b) prior knowledge may be much or little, but one is about to take in any case, or has just taken and is about to analyze, a very large sample, ^{and} sample results are of a certain sort (p. 67)

Bayesians admit possibility of "useful general-purpose statements of posterior probability" S&R p. 69 based on specified prior dists (e.g. rectangular): i.e. "confidence intervals," or "significance levels."

But this is admitted as useful only for reporting general-purpose scientific results. For decision, D ⁶⁸ must construct a definite prob dist (using, in part, ~~the info~~ the info provided; the "interval" cited is relevant only if D's prior dist is roughly rectangular, in which case he is saved computation). (e.g. D may have to decide on margin of safety, or to conduct further experiments).

I DENY that D always must, does, or can decide on "definite prior dist." He may want precisely an interval estimate; and it might be possible (?) to give him one.

R + D

Need to "insure against" "changes in values" (objectives); which may come from technological changes in feasibility, or changes in the nature of the external threat.

i.e. payoffs can change for given ~~set~~ outcomes. (Obvious with "value of info" — changes with each set of actions decision-makers are considering.)

More likely at lower levels of sub-optimizing. But still likely at highest national levels.

Use present theories on historical data to see how much uncertainty remains. (i.e. how much uncertainty we "should" recognize).
Could we have predicted major past events with current theories, + past info, or even with present info, or with both?

Problem here is "satisficing" attitude; if current objective outcome is "OK" in terms of present values, little impulse to make even cheap, major improvements. Yet if some thought were given to realistic estimates of likelihood of specific changes in values — or of a broad class of changes — improvement would seem much more worthwhile.

Moreover, some improvements (changes) in performance ~~as~~ ^{improvements in} ~~total~~ would lead to big _x payoffs — not under current payoff function — but under a broad class of different, possible payoff functions.

(In retrospect, one tends to say, "How did they fail to spend a few pennies on what was so obviously a valuable improvement?")

R+D : SM-41

R+D administrators acted according to my model; until the uncertainty of a useless result had been reduced (i.e. the likelihood of a useful result raised, say, to at least " $\frac{1}{n}$ ") they weren't interested; they preferred the less ambiguous minor improvement.

Thus, "bias" against the major innovation, even when its "best guess" "promise" is much greater (but ambiguous).

Tendency to underestimate likelihood of major surprise.

Bad, because in the range of "major surprise," the consequences of previous "error" are not merely a linear function of the "size of error," as they are with small or medium-sized errors (assuming there were "not too surprising" or with any errors of a sort considered "possible.")

The occurrence of the "impossible" event is likely to be far more disastrous than the occurrence of the "extremely unlikely" event.

(because of ^{total} neglect in planning).

Thus, tendency greatly to underestimate likelihood of disastrous failure of system. (And an opponent does well to probe & aim at accomplishing unplanned disaster in enemy; surprise is essential ingredient. How surprising an event must be to be unplanned-for is another question; opponents will differ).

or even $\alpha \approx 1$

Klein hopes to interest administrators by denaturing cheapness; I would emphasize possible value. Need both, (Reduce uncertainty on feasibility).
LET $P=1$ FOR "SMALL" POTENTIAL LOSSES (with big potential gains?)

If some "reasonable" opinions ~~from~~ see major improvements possible, take cheap steps to reduce uncertainty.

|| (Schelling-Hart stresses advantages and feasibility of achieving surprise. Consider why it is so possible, and so profitable (taking point of view of defender).

Operators underestimate value of big innovations, because their values (payoffs, objectives) are closely tied to their current operations, which reflect current objectives, which reflect current feasibilities.

Hence, big innovations (which would affect objectives, payoff function) don't seem as valuable (in terms of current payoff function) as minor innovations that don't change basic constraints, hence don't change payoff function. (The big changes, if introduced, would make these minor changes look "valueless." DIFFERENT PRICE SYSTEMS BEFORE AND AFTER MAJOR INNOVATION.

see p-16, SM-41

2 2

2

$10 \pm .0001$

Belief & action

We talk of "believing" or "not believing," when in fact we "believe to a certain degree." Whether a belief is "strong enough to act on" depends on the acts in question: i.e. the odds.

(Kaldor "doesn't believe" in SU surprise attack; but is willing to act "as if" there were a distinct possibility, e.g. $\frac{1}{10}$; take insurance.)

When "cheap" insurance is in question, issue should be relatively insensitive to probabilities (def is that the definition of "cheap"?).

To say you should act "as if" you could always compare two probes is not to say that you do always compare them in some other, intuitive sense. But it does say that if, when you "can" compare them in this other sense, you do act "as if" you could, then by ~~your~~ your actions should reveal no difference between your behavior (or state of mind) when you "can" compare them and when you "can't."

i.e. it "should" be impossible to tell from your behavior whether you "can" or "can't": you should always act "as if" you can.

Savage

The link between the intuitive and, behaviourist approaches to probability breaks down — in fact — in that area where probabilities are "vague" or "non-comparable" according to the intuitive approach. (Thus we can predict by intuitive approach when Savage axioms will be violated.)

Need a better behaviourist theory. (Smith?)

Smith:

	B	C			
Bit	x	-wx	Bit	1	-w
Reject	0	0	Reject	0	0

odds of w on B against C.

Bit	1	0	Bit	1-p	-p
			(with max payoff)		
Max payoff on bit	p	p			

Let $x = 1-p$. Then bit is $\frac{1-p}{1-p} \cdot x$ $\frac{-p}{1-p} \cdot x$

$$\frac{p}{1-p} = w^*$$

$$\text{or } 1 \times \frac{-p}{1-p} \cdot x$$

$$\text{or } x \quad -wx$$

~~In hitting on a single~~

Our intuition probes about the outcome of a single event, or the average outcome of a long sequence of events, may be very different from the probes "revealed" by our actions; and both may be different from "objective probes" (or anyone else's probes).

(PERSONAL _{but} vs PERSONAL _{guess}).

Unless one assents an empirical link; or requires a link, as a rational axiom (dubious).

(Any proposition establishing a link would have a different character from the props. internal to the theories. e.g., ~~might~~ would require "consistency" between what people do and what they say).

BUT: when confronted by a long sequence, or unambiguous evidence (e.g. assuming independence between trials, and no game), then

P_o and P_g should both be close ~~to P_o~~ ("to P_o ") for any one person, and for ^{"most"} ~~all~~ persons.

Smith

Let $p=0$, $\alpha=1$ (~~the~~) V^0 some interval of $0 \leq \text{prob}(E) \leq 1$

	E	$\bar{E}$
Bet on E	1	0
	p^*	p^*

p^* is the most I will pay for a prize of 1 on E (BAYES).

E	$\bar{E}$
0	1
q^*	q^*

Upper prob

q^* is the most I will pay for a prize of 1 on $\bar{E}$.

$q^* + p^* \leq 1$; otherwise, "incoherence"; bet can be made against me.

So $q^* \leq 1 - p^*$, $p^* \leq 1 - q^*$

Suppose $q^* + p^* < 1$. $q^* < 1 - p^*$ $p^* - (1 - p^*) < 1$

~~Then~~ $p^* + (1 - p^*) \geq q^* + p^*$ $2p^* < 2$ $p^* < 1$

Ex: $p^* = \frac{1}{4}$, $q^* = \frac{1}{4}$. Then I will not pay $\frac{1}{2}$ to bet on E , nor will I pay $(1 - \frac{1}{2}) = \frac{1}{2}$ to bet on $\bar{E}$. (i.e. $\frac{1}{2} > \text{prob}(E)$, and $(1 - \frac{1}{2}) > \text{prob}(\bar{E})$.)

(Usual assumption: $p^* + q^* = 1$. For any $0 \leq x \leq 1$, either

x is an acceptable price for a stake of 1 on E , or $(1-x)$ is.)

$$\text{Let } p_* = 1 - q^*$$

$$\text{Let } p = 0, \alpha = 0$$

1	0	0	1
p_*	p_*	q_*	q_*

$$p_* + q_* \leq 1$$

$$\text{Let } (1 - p_*) \text{ be } q^* \text{ and } (1 - q_*) \text{ be } p^*$$

$$\text{Then } p_* \leq (1 - q_*) = p^* \quad q_* \leq q^*$$

~~p_* is the most I will be willing to~~

$$\text{Interval of probs: } p_* \leq p \leq p^* \quad q_* \leq q \leq q^*$$

$$\text{Suppose } p_* + q_* < 1. \quad \text{Then } p_* < p^*, \quad q_* < q^*.$$

$$\text{Let } p' \text{ be such that: } p_* < p' < p^* \quad q_* < (1 - p') < q^*$$

$$\text{But } (1 - p') \quad -p' \quad \text{But } -p' \quad 1 - p'$$

$$\text{Don't } 0 \quad 0 \quad \text{Don't } 0 \quad 0$$

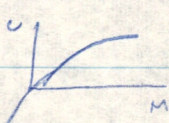
Prefer not to bet, either at $\frac{p'}{1-p'}$: 1 on E or against E at $\frac{1-p'}{1-p'}$: 1 against.

$$\text{e.g. if } p_* = \frac{1}{4}, \quad q_* = \frac{1}{4}, \quad p' = \frac{1}{2}, \quad \text{I won't bet at } 1:1 \text{ on } E \text{ or against } E.$$

(if I am allowed not to bet — Keynes, General Theory !)

I will but at $p' = \frac{1}{8}$, or $(1 - \frac{1}{8}) = \frac{7}{8}$

odds of $\frac{1}{7} : 1$ on E

Do Smith's propositions all still hold if utility is non-linear on money, but convex? 

I think so; but there will be a larger area within which one won't bet. i.e. if p_* is the most I will give for a stake of 1, I will not give more than $\frac{p_*}{1} \cdot X$ for a stake of X (in fact, I may not give as much as $\frac{p_*}{1} \cdot X$; statements about upper bounds remain "safe", though they may not be l.v.b.'s as stakes increase.) So: established interval of probs using ~~moderate~~^{big} amounts of money, and interval will $\alpha(p^* - p_*) \leq -\frac{1}{2}$ enclose "true" probability interval; use small or moderate amounts for greater ^{precision} "accuracy" of prob estimate.

$$\beta \quad \alpha p^* + (1-\alpha)p_* + \alpha q_*^* + (1-\alpha)q_* \leq 1$$

$$\alpha p^* + 1 - \alpha p_* + \alpha q_*^* + 1 - \alpha q_* \leq 1$$

$$\alpha p^* + 1 - \alpha p_* + \alpha(1-p_*) + 1 - \alpha(1-p^*) = \alpha p^* + 1 - \alpha p_* + \alpha - \alpha p_* + 1 - \alpha + \alpha p^*$$

$$2\alpha p^* - 2\alpha p_* + 2 \leq 1 \quad \cancel{2\alpha p^* + 2} \quad \alpha p^* - \alpha p_* \leq -\frac{1}{2}$$

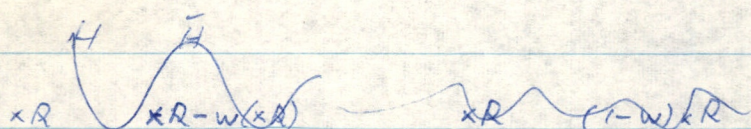
My Sure-thing Principle:

$$I \quad \geq \quad b \quad c$$

$$II \quad b \quad \geq \quad c$$

If $I \succ II$ for some $C = C'$, then either $I \succ II$ for all $C \geq C'$
or $I \succ II$ for all $C \leq C'$.

In particular, ~~if~~ relation of I to II should be constant for
all C : $C \geq \max(a, b)$ and for all C : $C \leq \min(a, b)$.



On Smith: Commentator focuses on his discussion of decision problem
in deciding between two gambles when probes overlap. Smith suggests
that choice will be stochastic, violating P1. What is more important is that
one might consistently prefer not to bet at certain odds, violating P2 rather
than P1.

Imagine von Neuman-Morgenstern axioms with probe intervals instead of
point probes — get utility intervals. ~~Then use~~

Or use known utilities to get probe intervals.

	E	$\bar{E}$	
I	2	-1	I is a bet on E at 2:1 on E.
II	0	0	II is a bet against E at 1:2 against E (2:1 on E).
III	-2	1	on $\bar{E}$ at 1:2 on $\bar{E}$ (2:1 against).

Savage: Can't have both $II \succ I$ and $II \succ III$.

$$III \approx -(I) \quad u(III) = -1 \cdot u(I)$$

Multiply by (-1) reverses ordering (unless both = 0); so if $II \succ I$, $-(I) \succ II$.

Assume there are utilities.

$$II \succ I \Rightarrow p \cdot 2 + (1-p)(-1) < 0$$

$$3p < 1$$

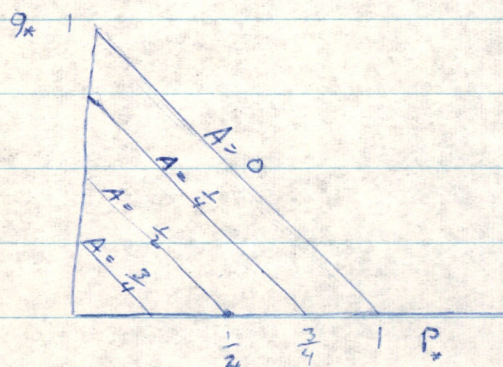
$$p < \frac{1}{3}$$

$$II \succ III \Rightarrow -2p + (1-p) \cdot 1 < 0$$

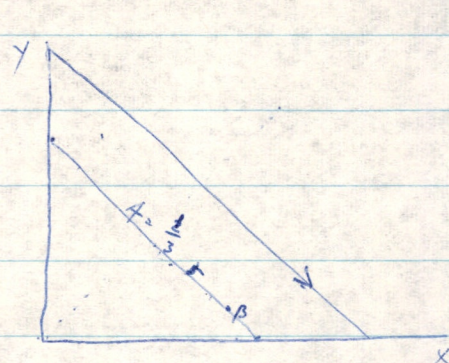
$$-3p < -1$$

$$p > \frac{1}{3} \quad \text{contradiction}$$

There exists no p such that: ~~I~~ $p < \frac{1}{3}$ and $p > \frac{1}{3}$, i.e. such that $u(I) > 0$ and $u(III) > 0$.



Each "ambiguity line" corresponds to gambles such that $x_\alpha + y_\alpha = A < 1$, where $A = 1 - x_\alpha - y_\alpha$ is an "index of ambiguity", since interval between upper and lower probs of α and of $\bar{\alpha} = A$.



$$q^* - q_\alpha \quad p^* - p_\alpha$$

$$A = (1 - x_\alpha) - y_\alpha = (1 - y_\alpha) - x_\alpha$$

For points on a ~~given~~ line, $A=0$, order of likelihood of α corresponds 1:1 to position on line, increasing to the right.

For points on another line, e.g. $A = \frac{1}{3}$, β to the right of α implies both the upper prob of $\beta > p^*(\alpha)$ and $p_*(\alpha) > p_*$; which is a necessary condition for $p(\beta) > p(\alpha)$.

$x_\beta - x_\alpha > \frac{1}{3} \Rightarrow p_*(\beta) > p^*(\alpha)$, which is a sufficient condition for $p(\beta) > p(\alpha)$.

(it will also be true that ~~$x_\beta - x_\alpha$~~ $y_\alpha - y_\beta > \frac{1}{3}$)

since $x_\beta - x_\alpha = y_\alpha - y_\beta$, since $x_\alpha + y_\alpha = x_\beta + y_\beta = A$.

$$x_\alpha + y_\alpha = x_\beta + y_\beta = A = \frac{2}{3}$$

Suppose $x_\alpha \geq 1 - y_\beta$ (i.e. $p_*(\alpha) \geq p^*(\beta)$)

and $1 - y_\alpha \leq y_\beta$

Let $x_\alpha = 1 - y_\beta$, $1 - y_\alpha \leq y_\beta$, or $y_\alpha \leq 1 - y_\beta = x_\alpha$

$$1 - y_\beta + 1 - y_\alpha \leq \frac{2}{3}$$

$$2 - 2y_\beta \leq \frac{2}{3}$$

$$1 - y_\beta \leq \frac{2}{3}$$

$$-y_\beta \leq \frac{1}{3}$$

$$y_\beta \geq \frac{1}{3}$$

So $x_\alpha \geq \frac{2}{3}$, $y_\alpha \leq \frac{2}{3}$

Suppose $x_\alpha + y_\alpha = x_\beta + y_\beta = (1-A) = \frac{2}{3}$

Suppose $x_\alpha - x_\beta \geq y_\beta - y_\alpha \geq \frac{1}{3} = A$

$$x_\alpha \geq \frac{1}{3} + 1$$

Then $p_*(\alpha) = x_\alpha$ $1 - x$

$$x_\beta = 1 - A - y_\beta \quad y_\beta = 1 - A - x_\beta$$

$$1 - y_\beta = -(1 - A - x_\beta) + 1 = A + x_\beta$$

If $x_\alpha > A + x_\beta$, then $x_\alpha > 1 - y_\beta$; i.e. $p_*(\alpha) > p^*(\beta)$

and $1 - y_\alpha < y_\beta$, i.e. $p^*(\alpha) < p_*(\beta) \Rightarrow p_*(\alpha) > p_*(\beta)$

$$1 - x_\alpha < y_\beta$$

Proof:

1) Suppose $x_\alpha + y_\alpha = x_\beta + y_\beta = (1-A)$ (α & β are on same "antiquity line.")

2) Suppose $x_\alpha - x_\beta = y_\beta - y_\alpha > 0$ (α to right of β)

3) Then $x_\alpha > x_\beta$, or $p_*(\alpha) > p_*(\beta)$

4) and $(1-y_\alpha) > (1-y_\beta)$, or $p^*(\alpha) > p^*(\beta)$

which is necessary for $p(\alpha) > p(\beta)$ (Koopman)

5) Suppose $x_\alpha - x_\beta = y_\beta - y_\alpha \geq A > 0$.

6) Then $x_\alpha \geq A + x_\beta$. (by 5)

7) But $y_\beta = (1-A) - x_\beta$ (by (1))

8) So $(1-y_\beta) = -(1-A-x_\beta) + 1 = A + x_\beta$

9) So $x_\alpha \geq (1-y_\beta)$ (6) + (8) $\Rightarrow p_*(\alpha) \geq p^*(\beta)$

likewise, ~~so $y_\alpha = 1-A-x_\alpha$ (1)~~

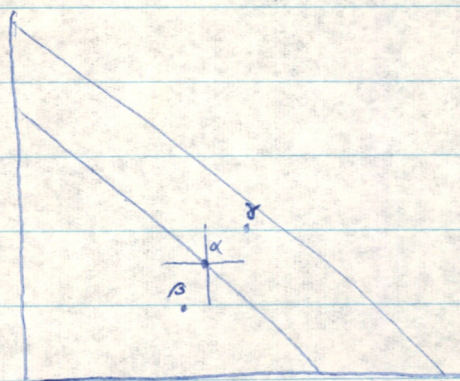
~~so $(1-y_\alpha) = 1-(1-A-x_\alpha) = A+x_\alpha$ But $x_\alpha = 1-A-y_\alpha$ (1)~~

But $y_\beta \geq A+y_\alpha$ (by 5) = ~~so $(1-x_\alpha) = A+y_\alpha$~~

~~$y_\beta \geq 1-A-x_\beta \geq A+y_\alpha$ to $y_\beta \geq (1-x_\alpha)$~~

or $p_*(\beta) \geq p^*(\alpha)$

SO $p(\alpha) \geq p(\beta)$

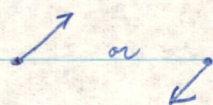


β southwest of α : $x_\beta < x_\alpha$ and $y_\beta < y_\alpha$.

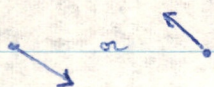
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 ~~$x_\alpha > x_\beta$~~ $x_\alpha > x_\beta$ $p_*(\alpha) > p_*(\beta)$
 $(1 - y_\alpha) < (1 - y_\beta)$ $p^*(\alpha) < p^*(\beta)$

Interval for β encloses interval for α .

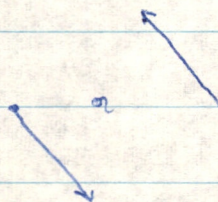
δ Northeast of α : interval for δ is enclosed by interval for α .



Nested intervals; "non-comparable"?

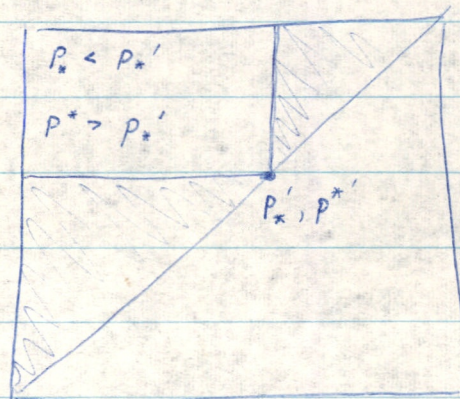
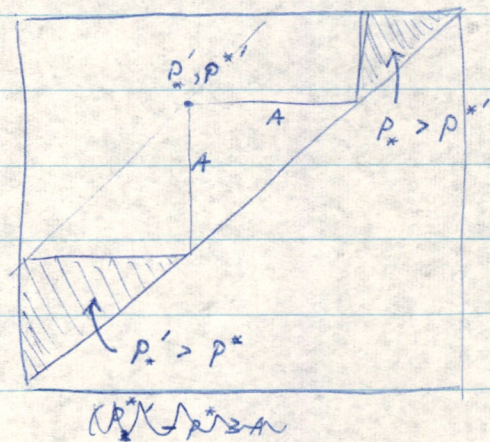
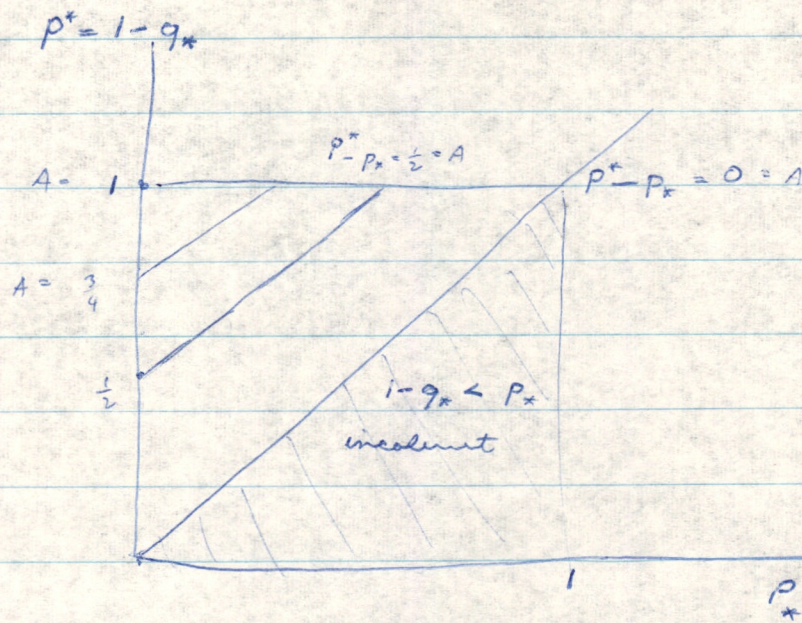


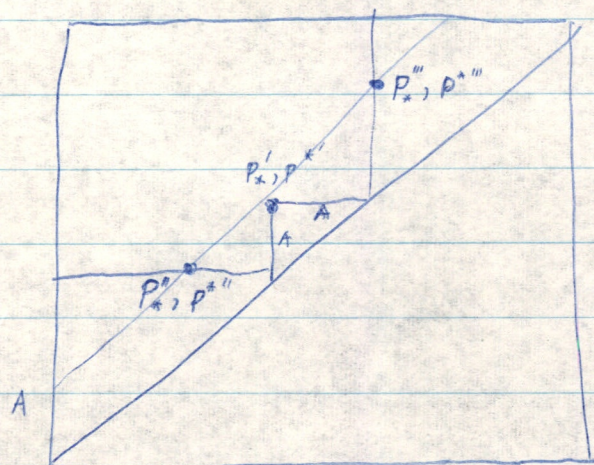
Overlapping intervals; may be comparable, depending on decision rule, or rule for reporting.



"far enough": Non-overlapping intervals: probes comparable.

How far it is necessary to ~~affair~~ go to get comparability depends on level of "ambiguity," i.e. size of intervals. On line $A=1$, all probes are comparable. Further into space (toward origin), must separate by greater interval ($= A$).

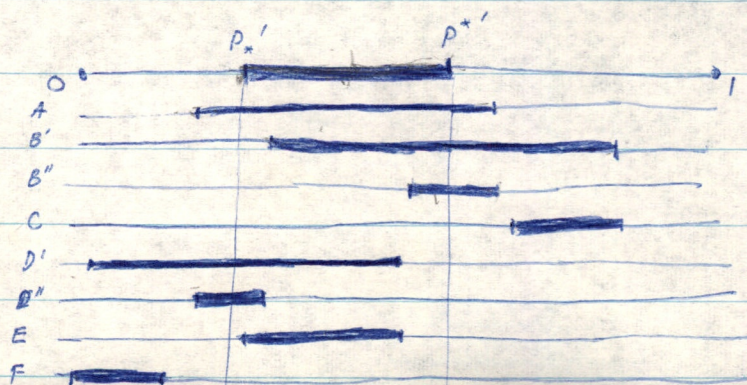
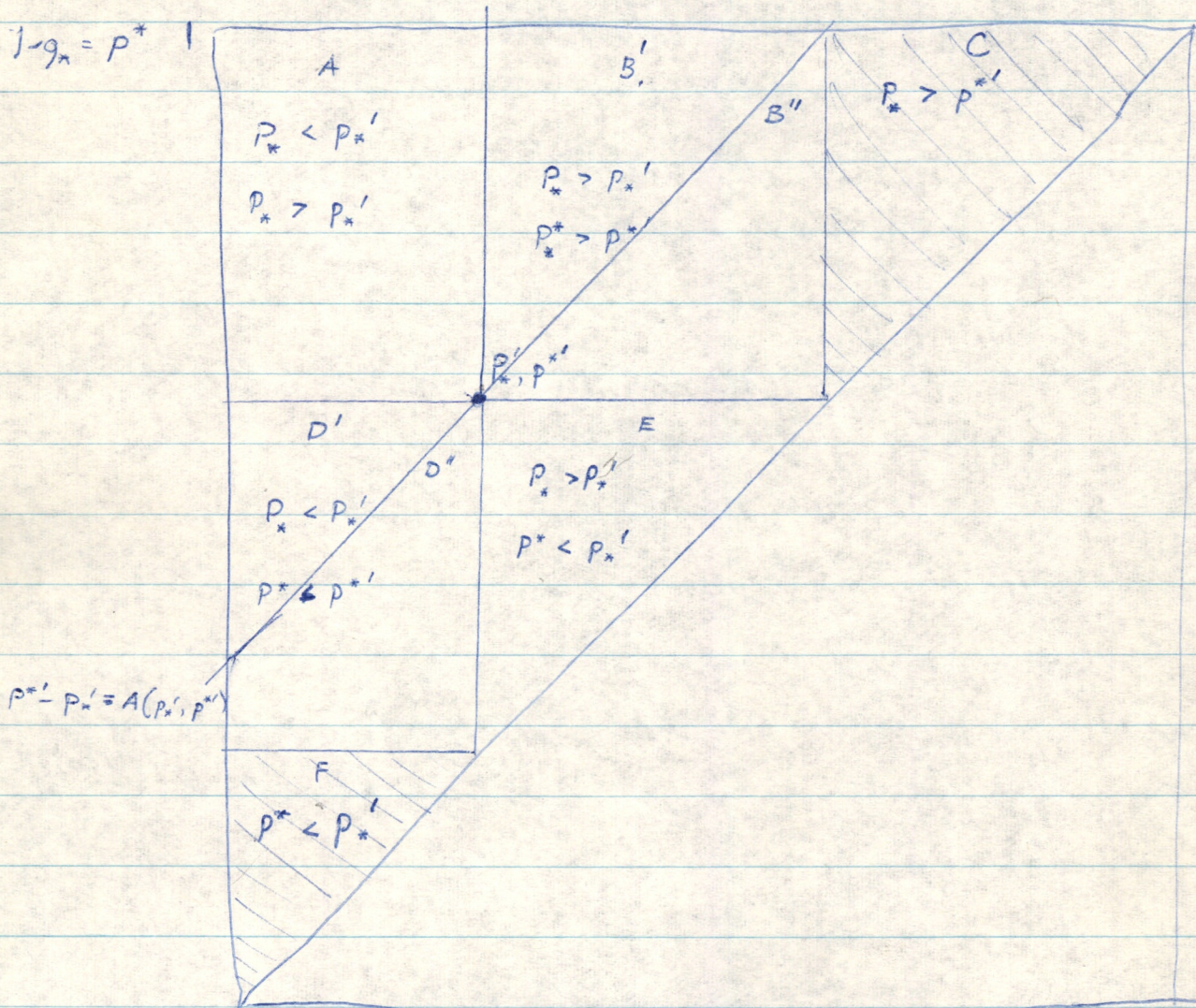




$$P_{*'}^{'} - P_{*}^{''} = A$$

$$P_{*}^{''} - P_{*'}^{'} = A$$

E	$\bar{E}$	E	$\bar{E}$	
1	0	0	1	
p_*	p_*	q_*	q_*	$p^* = 1 - q_*$



Should also have F', C'

$$\begin{array}{cc} 2 & -1 \\ 0 & 0 \\ -2 & 1 \end{array}$$

	H	T		E	$\tilde{E}$
Utilities imply: I	2	0	2	0	I 2 -2
II	1	1	0	2	II -2 2
III	0	-2	1	1	III 0 0
IV	-1	-1	1	-1	IV 2 0
V	0	0	-1	1	V 0 2
VI	2	-2	0	0	VI 1 1
VII	-2	2			VII 1 -1
					VIII -1 1
					IX 0 -2
					X -2 0
					XI -1 -1
					XII 2 2
					XIII -2 -2

$$I \circ II \Rightarrow IV \circ V, VII \circ VIII, IX \circ X$$

$$+ \text{assignment of } 0 \Rightarrow I \circ II \circ III \circ VII \circ VIII \circ (IX, X; \frac{1}{2})$$

$$+ \text{assignment of } 1 \Rightarrow IV \circ V \circ VI$$

$$+ \text{assignment of } -1 \Rightarrow IX \circ X \circ XI$$

$$\begin{array}{ccccc} & & E & \tilde{E} & \\ \text{Suppose } A & 2 & -1 & & A \circ B \\ & B & 0 & 0 & \end{array}$$

$$\text{Then } 2 \quad -1$$

+

	E_1	E_1'	E_2	E_2'
I	2	-1	0	0
II	-2	1	0	0
III	0	0	2	-1
IV	0	0	-2	1
V	0	0	0	0
VI	2	2	-2	-2
VII	-2	-2	2	2
VIII	2	2	0	0
IX	0	0	2	2
X	1	1	1	1
XI	0	0	-2	-2
XII	-2	-2	0	0
XIII	-1	-1	-1	-1
XIV	1	1	-1	-1
XV	-1	-1	1	1

Suppose given: $\text{VII} \prec \text{VI} \prec \text{XIV} \prec \text{XV} \prec \text{IX}$ } $\text{pr } (E_1 \cup E_1') = \text{pr } (E_2 \cup E_2') = \frac{1}{2}$
 $\text{VIII} \prec \text{IX} \prec \text{X}$ } $\text{then arbitrary assignment of}$
 $\text{XI} \prec \text{XII} \prec \text{XIII}$ } $2 \text{ and } -2, 1, 0, -1 \text{ are "utilities"}$

Suppose further (b): $\text{I} \not\prec \text{II}$

Prove: $\text{III} \prec \text{IV} \prec \text{V}$

or: Suppose (c): $\text{III} \prec \text{V}$

Prove: $\text{III} \prec \text{IV}$ ($\text{I} \prec \text{II} \prec \text{V}$)

(d)
 or suppose: $\text{II} \prec \text{III}$ prove from (a): not both $\text{II} \prec \text{III}$ and $\text{V} \prec \text{IV}$; and not both $\text{IV} \prec \text{I}$ and $\text{V} \prec \text{II}$

Proof of (d). Suppose I, III . To prove: II, V . Given (a).

$$\begin{array}{ccccccccc}
 \text{I} & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 \\
 \downarrow & & & & & & & & \\
 \text{III} & 0 & 0 & 2 & -1 & 0 & 0 & 0 & 0
 \end{array}$$

Koopman: Axioms

T sub: $a/b < b/k, b/k < c/l \Rightarrow a/b < c/l$ Transitivity

Antisymmetry: If $a/b < b/k$, then $\sim a/b > \sim b/k$

(Thus, betting quotients don't always correspond to intuitive probabilities, since people don't always obey Savage axioms. But certain pairs of betting quotients may conform to "upper & lower probabilities", given coherence.)

R ~~Reflexivity~~ Reflexivity: $a/b < a/b$.

R + T define a partial ordering on propositions.

Def: π -scale (283)

Th. 17: $p_*(a, b) + p^*(\sim a, b) = 1$ ($p_*(a, b) + p_*(\sim a, b) \leq 1$).

Th. 18: i) $p^*(a, \vee a_2, b) \leq p^*(a, b) + p^*(a_2, b)$.

ii) If $a, a_2, b = 0$, then $p_*(a, \vee a_2, b) \geq p_*(a, b) + p_*(a_2, b)$

Th. 19. If a, b, c are any propositions for which $a < b < c$ ($b, c \neq 0$), then

p_*

at

Good: $P_*(F|H) = f_*$, $P^*(F|H) = f^*$, $P_*(E|H) = e_*$, $P^*(E|H) = e^*$,
 $P_*(E \cdot F|H) = g_*$.

$g_* \geq e_* f_*$, $g^* \geq e_* f^*$, $g^* \geq e^* f^*$, $g_* \geq e^* f_*$, $g_* \leq e_* f^*$,

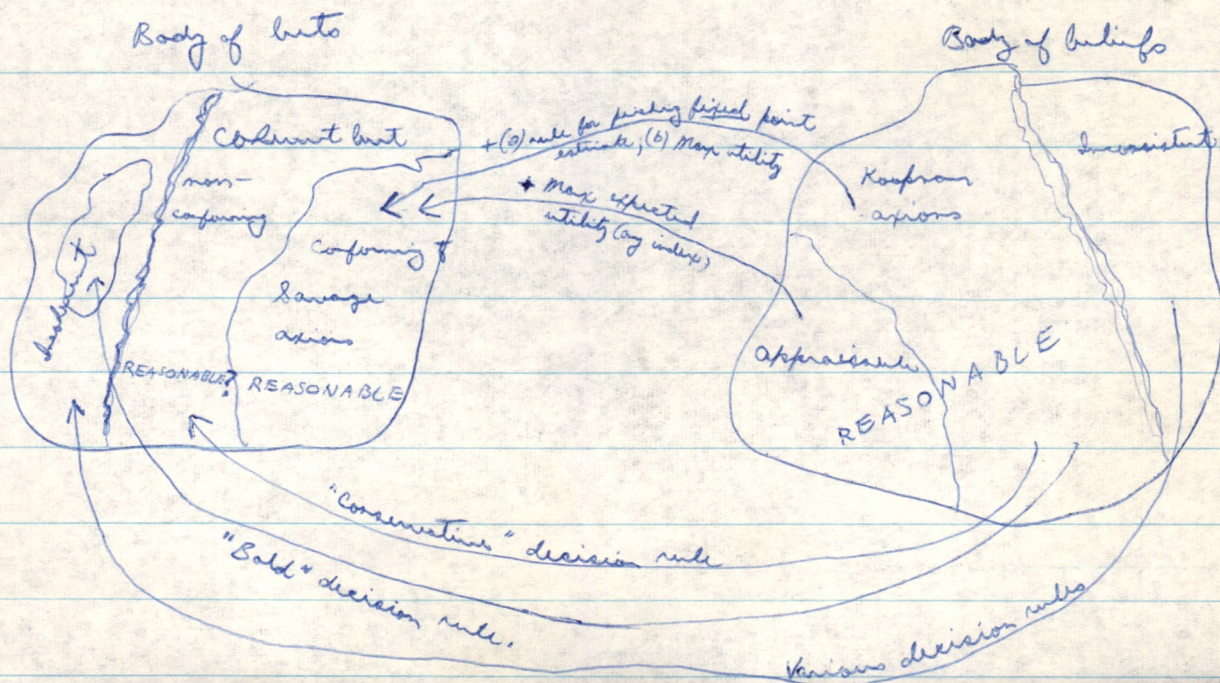
$g_* \leq e^* f_*$, $g^* \leq e^* f^*$.

Wolf GR-44764

Dugov EX-75864

From behavior conforming to the Savage axioms, you can't infer internal estimates of intuitive probability; no way to make such notions as "upper and lower" probs meaningful (unless "small" violations of the axioms are allowed, leading to small intervals). You can't predict appraisability, or discriminate between small and large intervals.

Do you care? Not if you are interested only in predicting the behavior

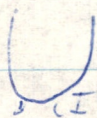


Assume utility is convex (MU is decreasing) over region $0-S$.
Then the upper and lower probs for an event E based on bets involving the state S will enclose the upper + lower probs for bets with any state $0 \leq s \leq S$. If $P_* = P^*$ for S , great.

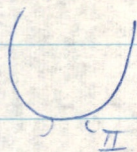
The upper + lower probs "revealed" in betting should be enclosed in the "intuitive" upper and lower probs; they will depend on the latter, + the "best guess", + p, α .

Thus, if forecaster gives internal probs based on intuition, he should also be able to give a decision rule from which, given payoffs, we could predict his betting behavior, thus "checking" his assertion of intuitive probs (assuming "correctness" of decision rule and payoffs).

What we want from forecaster are his internal estimates and best guesses; we'll supply our own payoffs and decision rule. But we might want his decision rule and payoffs to check his estimates against his actual betting behavior.



5R, 4B



50R, 50B

	R_I	B_I		R_{II}	B_{II}	
I	1	0	I : II	III	1	0
II	$\frac{1}{2}$	$\frac{1}{2}$		IV	$\frac{1}{2}$	$\frac{1}{2}$
V	0	1		V	0	1
or VII	-1	0		IX	-1	0
VIII	$-\frac{1}{2}$	$-\frac{1}{2}$		X	$-\frac{1}{2}$	$-\frac{1}{2}$

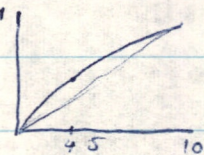
I : II should $\Rightarrow$ VII : VIII

("But guess" y^0 is no longer $\frac{1}{2}, \frac{1}{2}$; he is no longer indifferent between I and II).

Love of gambling, excitement, can be identified even when bets are in utilities; can appear as love or hatred of ambiguity.

Suppose $U(10) = 1$, $U(80) = 0$, $U(44) = \frac{1}{2}$; i.e. $\begin{matrix} H & T \\ I & 10 & 10 \\ II & 44 & 44 \end{matrix}$ I : II.

By this utility function:



we express "dislike of gambling" or

fact that I (gamble) is preferred to II (sure thing) only if $E_M(I) - E_M(II) > 1 > 0$

Now suppose $\begin{matrix} R_I & B_I \\ I & 10 & 0 \\ II & 4 & 4 \\ III & 0 & 10 \end{matrix}$. I < II. To produce I : II, have to make

$\begin{matrix} R_I & B_I \\ I & 10 & 0 \\ II & 3 & 3 \end{matrix}$ i.e. $E_M(I) - E_M(II) > 2 > 1 > 0$.

1.

~ meaningfulness of statements: The prob of heads is
Nixon's election is

I don't know the prob of

I am more sure of $A > B$ than of $C > D$.

When ~~Why~~ Should Bayesian & Statisticians Not Maximize Moral Expectation?

Insurance:

Assume all probabilities "known."

Compare Coherent Reports on Follies

I
Insurance action vs non-insurance action II:

(a) Presumably I doesn't dominate II. (b) But $U(I) > U(II)$, if insurance is desirable. From (a), there must be some subset of events E for which II is better than I. Insurance is a bet against E : on $\bar{E}$. $I > II \Leftrightarrow$ (the expected value of I given $\bar{E}$) $\Rightarrow$ (the expected value of II given $\bar{E}$) $>$ (the expected value of II given E) $-$ (the expected value of I given E). In particular, insurance action I has a sure outcome, or a higher minimum outcome.

	E p	$\bar{E}$ $1-p$	
II	10	0	0 — 4
I	4	4	6 — 0

$$II < I \Leftrightarrow p(10-4) < (1-p)(4-0)$$

I is a "gamble" when opportunity costs, or regrets, are considered.

"Necessarists" and "abjectivists" also are interested in "complete ignorance" case, because their rule for this gives equal probs, and leads to determinate probs from then on."

(Savage, Keynes... would allow "inherited" probs even in that case.)